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DISCUSSION OF TORSION OF I-TYPE AND H-TYPE BEAMS

(Published in August, 1952)

By Kurt H. Gerstle, and John E. Goldberg

STRUCTURAL DIVISION

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Proceedings—Vol. 79 May, 1953 Separate No. D-145

DISCUSSION

Kurt H. Gerstle,[10] J. M. ASCE.—Engineers concerned with the analysis of members that resist an applied torque by both twisting and bending of the fibers will find this paper of interest and value. The outline of the stress analysis, as well as the summary of expressions for the twisting of beams subject to various boundary conditions, will facilitate the application to practical cases of the theory of torsional bending.

The writer believes that the main obstacle to the widespread use of the method is the time consuming indeterminate solution of the beam with restrained ends subject to a torque applied at an intermediate point. Obviously, the author has realized this fact and, therefore, has presented three different methods of solution for this fundamental problem. The application of the Müller-Breslau principle would greatly simplify this solution, and, therefore, might help in the application of the torsional bending theory to practical design work. The writer will demonstrate the application of this principle.

According to the Müller-Breslau principle, the influence line for a certain stress element will be proportional to the deflected shape of the structure which results when the stress element desired is replaced by a displacement in the sense and direction of this stress element.[11] The value of the influence ordinate at any point then will be given by the negative ratio of the deflection at that point to the displacement induced.

The problem at hand is the determination of the influence line for a reactive torque at one end produced by a torque applied at any point of the member. Therefore, the induced displacement will be an angle of twist at one end, and the deflection proportional to the influence ordinate will be given by the corresponding angle of twist at any section. Accordingly, solving for the reactive torque $T_{(x=L)}$ due to an applied torque T—and using Eq. 8—

$$T_{(x=L)} = \frac{\phi}{\phi_{(x=L)}} T = \frac{A_1 \sinh \frac{x}{a} + A_2 \cosh \frac{x}{a} + \frac{T}{C} x + A_3}{A_1 \sinh \frac{L}{a} + A_2 \cosh \frac{L}{a} + \frac{T}{C} L + A_3} T \ldots (45)$$

For a beam with both ends rigidly fixed against twisting, $\omega_0 = 0$, and $\omega_L = 0$. In this case, the constants A_1, and A_2, and A_3 are

$$A_1 = -a\frac{T}{C}; \quad A_2 = \frac{T\,a}{C} \tanh \frac{L}{2\,a}; \quad A_3 = -\frac{T\,a}{C} \tanh \frac{L}{2\,a}$$

Putting these constants into Eq. 45,

$$T_{(x=L)} = -\frac{\frac{x}{a} - \sinh \frac{x}{a} + \tanh \frac{L}{2\,a}\left(\cosh \frac{x}{a} - 1\right)}{\frac{L}{a} - \sinh \frac{L}{a} + \tanh \frac{L}{2\,a}\left(\cosh \frac{L}{a} - 1\right)} T \ldots\ldots\ldots (46)$$

Note.—This paper by John E. Goldberg was published in August, 1952, as *Proceedings-Separate No. 145*. The numbering of footnotes and equations in this Separate is a continuation of the consecutive numbering used in the original paper.

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[11] "Elementary Structural Analysis," by J. B. Wilbur and C. H. Norris, McGraw-Hill Book Co., Inc., New York, N. Y., 1948, p. 450.

This expression represents the influence line for the reactive torque at the end where x equals L produced by a torque T applied to any point x of the fixed-ended beam.

Substituting values for the beam of the author's first example (see under the heading, "Illustrative Examples") in Eq. 46,

$$\left.\begin{array}{l} T_2 = \dfrac{\dfrac{60}{51.80} - \sinh\dfrac{60}{51.80} + \tanh\dfrac{180}{103.60}\left(\cosh\dfrac{60}{51.80} - 1\right)}{\dfrac{180}{51.80} - \sinh\dfrac{180}{51.80} + \tanh\dfrac{180}{103.60}\left(\cosh\dfrac{180}{51.80} - 1\right)} T = 0.268\ T \\ \text{Similarly,} \\ \qquad\qquad T_1 = 0.732\ T \end{array}\right\} \quad \ldots(47)$$

If, in addition to the restraint produced by the fixed-ended condition, prescribed warping angles ω_1 and ω_2 and a differential displacement ϕ_e are imposed, their effect on the reactive torques can be computed by the procedure implied in Eq. 39:

$$- \phi_e = \frac{2\,T}{C}\left(\frac{L}{2} - a \tanh\frac{L}{2\,a}\right) + \left(\frac{\omega_1 - \omega_2}{h}\right) a \tanh\frac{L}{2\,a};$$

or

$$\frac{T_1}{C} = -\frac{T_2}{C} = \frac{-\phi_e - \left(\dfrac{\omega_1 - \omega_2}{h}\right) 48.6}{2\,(90 - 48.6)}$$

$$= \mp\, 0.0121\,\phi_e \pm 0.589\left(\frac{\omega_2 - \omega_1}{h}\right) \quad \ldots(48)$$

Superposing the effects shown in Eqs. 47 and 48 leads to Eqs. 37. In the writer's opinion, the foregoing computations involve considerably less time and effort than the methods demonstrated by the author.

JOHN E. GOLDBERG,[12] M. ASCE.—The writer is grateful for Mr. Gerstle's appraisal of his paper and for the interest which has moved him to suggest an additional method for applying Eq. 8 to the solution of the basic type of problem. The writer believes that the obstacle to widespread use of the theory is not, as Mr. Gerstle feels, the time involved, but rather an unawareness of the existence of the actual problem and of the method of formulation. A primary purpose of the paper was to stimulate interest in, and to cause a wider understanding of, the phenomenon of warping. The methods of solution which were presented were not presumed to constitute a complete and closed group of procedures since the basic equations may be combined in several ways to devise various alternative methods.

Mr. Gerstle's suggested use of the Müller-Breslau principle, in conjunction with the given equations, to obtain an influence line for reactive torque at the

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end of the member is quite correct and useful. However, what is required is not only the reactive torque, but also the warping moments, the flange shears, and the various stresses. This means that warping conditions must be evaluated at a sufficient number of points, so as to particularize the arbitrary constants of Eq. 8. Alternatively, Eq. 18 or Eq. 21 may be used in the computation of stresses. Since ω_0 is treated as one of the unknowns in Eq. 22, this approach yields the necessary data for completing the stress analysis. The trigonometric series procedure has the advantage of not requiring explicit computation of intermediate warping.

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